

Diploma Mathematics formulae(25SC11T)

MATRICES

A matrix is a rectangular arrangement of numbers in rows and columns.

Order of a matrix = (Number of rows) \times (Number of columns). ($m \times n$)

Example: A (2×3) matrix has 2 rows and 3 columns.

Example:

$A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 1 & 5 \end{bmatrix}$ is a 2×3 matrix.

Types of Matrices:

Row Matrix: Only one row.

Example: $[1 \ 2 \ 3]$

Column Matrix: Only one column.

Example: $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Zero (Null) Matrix: All elements are zero.

Example: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Square Matrix: Rows = Columns.

Example: $\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$

Diagonal Matrix: Non-diagonal elements are

zero. Example: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 4 \end{bmatrix}$

Scalar Matrix: Diagonal elements equal.

Example: $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Unit (Identity) Matrix:

Diagonal elements = 1, others = 0.

Example: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Algebra of Matrices:

Scalar Multiplication: $k \times A$ = Each element multiplied by k .

Example: $2 \times \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 8 \\ 4 & 6 \end{bmatrix}$

Transpose of a Matrix (A^T): Rows become columns, columns become rows.

Example: $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 5 \end{bmatrix} \rightarrow A^T = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 4 & 5 \end{bmatrix}$

Addition & Subtraction (2×2 matrices):

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$ then:

$$A + B = \begin{bmatrix} 6 & 5 \\ 5 & 5 \end{bmatrix}$$

$$A - B = \begin{bmatrix} -4 & -1 \\ 1 & 3 \end{bmatrix}$$

Multiplication of 2×2 matrices:

If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$, then:

$$\begin{aligned} A \times B &= \begin{bmatrix} (1 \times 3) + (2 \times 5) & (1 \times 1) + (2 \times 2) \\ (3 \times 3) + (4 \times 5) & (3 \times 1) + (4 \times 2) \end{bmatrix} \\ &= \begin{bmatrix} 3 + 10 & 1 + 4 \\ 9 + 20 & 3 + 8 \end{bmatrix} \\ &= \begin{bmatrix} 13 & 5 \\ 29 & 11 \end{bmatrix} \end{aligned}$$

DETERMINANTS

A determinant is a scalar value calculated from a square matrix.

2×2 Determinant:

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then:

$\det(A) = ad - bc$ (Product of Principle diagonals) \times (Product of secondary diagonals)

Example:

If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, then

$$\begin{aligned} \det(A) = \Delta &= \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} \\ &= (2 \times 5) - (3 \times 4) \\ &= 10 - 12 \\ &= -2 \end{aligned}$$

Cramer's Rule

$$a_1x + a_2y = c_1$$

$$b_1x + b_2y = c_2$$

Then formula find the x and y

Steps to find x and y

$$\text{Step 1 : find } \Delta = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$\text{Step 2 : find } \Delta_x = \begin{vmatrix} c_1 & a_2 \\ c_2 & b_2 \end{vmatrix}$$

$$\text{Step 3 : find } \Delta_y = \begin{vmatrix} a_1 & c_1 \\ b_1 & c_2 \end{vmatrix}$$

Step 4 : Find x and y by using the formula

$$x = \frac{\Delta_x}{\Delta} \quad \text{and} \quad y = \frac{\Delta_y}{\Delta}$$

Finding the adjoint of a matrix

For the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Step 1 : Interchange the Principle diagonal elements

$$\text{adj}A = \begin{bmatrix} 4 & \\ & 1 \end{bmatrix}$$

Step 2 : change the sign of the secondary diagonal elements

$$\text{adj}A = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

Inverse of a matrix

$$A^{-1} = \frac{\text{adj}A}{|A|}$$

Characteristic Equation

If A is the square matrix, then

Characteristic Equation is defined as

$$|A - \lambda I| = 0$$

CE can be obtained by

$$\lambda^2 - (\text{tra}A)\lambda + \Delta = 0$$

Vectors

Vectors representation

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = (a_1, a_2, a_3)$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = (b_1, b_2, b_3)$$

Modulus of a Vector

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Unit vector

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$$

Addition of two vectors

$$\text{If } \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = (a_1, a_2, a_3)$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = (b_1, b_2, b_3) \text{ then}$$

$$\vec{a} + \vec{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

Similarly

$$\vec{a} - \vec{b} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$$

Position vector of a Point

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$\vec{BC} = \vec{OC} - \vec{OB}$$

$$\vec{AC} = \vec{OC} - \vec{OA}$$

Dot Product or Scalar Product

$$\text{If } \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = (a_1, a_2, a_3)$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = (b_1, b_2, b_3) \text{ then}$$

$$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$$

If $\vec{a} \cdot \vec{b} = 0$ then the two vectors are said to be orthogonal or Perpendicular

Projection of a vector on the other vector

Projection of a vector \vec{a} on \vec{b}

$$\text{is defined as } = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

Projection of a vector \vec{b} on \vec{a}

$$\text{Is defined as } = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$$

Cosine of the angle between the two vectors defined as

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

Work done is defined as

$$W = \vec{f} \cdot \vec{s}$$

Where f is the force and S is the displacement

Trigonometric formulae

Radian to degree conversion and vice versa

$x \text{ radian} = x \frac{180}{\pi} \text{ degree}$		$x \text{ degree} = \frac{\pi}{180} \times x \text{ radians}$	
$\frac{\pi}{2}$	$\frac{\pi}{2} \times \frac{180}{\pi} = 90^\circ$	90°	$90^\circ \times \frac{\pi}{180} = \frac{\pi}{2}$
$\frac{\pi}{4}$	45°	45°	$\frac{\pi}{4}$
$\frac{\pi}{12}$	15°	15°	$\frac{\pi}{12}$
$\frac{5\pi}{12}$	75°	75°	$\frac{5\pi}{12}$

T - Values of standard angles

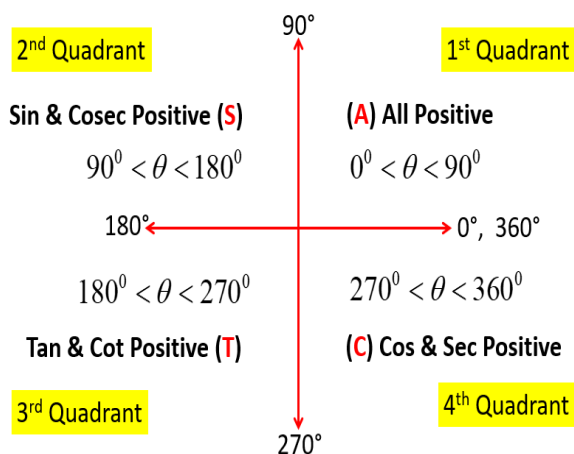
	0°	30°	45°	60°	90°	180°	270°	360°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	$-\infty$	0
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	∞	0	∞
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	1	∞	1
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	∞	-1	∞

Trigonometric ratios of Allied angles (without proof)

Trigonometric ratios of allied angles, when the sum or difference of two angles is either zero or a multiple of 90° . For example 30° and 60° are allied angles because their sum is 90° .

The angles $-\theta$, $90^\circ \pm \theta$, $180^\circ \pm \theta$, $360^\circ \pm \theta$ etc. are angles allied to the angle θ , if θ is measured in degrees. However, if θ is measured in radians, then the angles allied to θ are $-\theta$, $\frac{\pi}{2} \pm \theta$, $\pi \pm \theta$, $2\pi \pm \theta$ etc.

Using trigonometric ratios of allied angles we can find trigonometric ratios of angles of any magnitude.



Note:

Easy steps to find the allied angle

Step 1(Negativiting)

If it's a t ratio of negative angle change it to positive by using the following table

$\sin(-\theta) = -\sin\theta$	$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$
$\cos(-\theta) = +\cos\theta$	$\sec(-\theta) = \sec\theta$
$\tan(-\theta) = -\tan\theta$	$\cot(-\theta) = -\cot\theta$

Step 2(Splitting)

Express the given angle as $90 \pm \theta$, $180 \pm \theta$, $270 \pm \theta$ and $360 \pm \theta$, Check which group does it belongs to (we have divided into two groups 1. Odd group($90 \pm \theta$ and $270 \pm \theta$) and 2. Even group($180 \pm \theta$, $360 \pm \theta$)

Step 3(Grouping)

If it

Table : 4.1

Angle/ Function	$-\theta$	$90^\circ - \theta$ or $\frac{\pi}{2} - \theta$	$90^\circ + \theta$ or $\frac{\pi}{2} + \theta$	$180^\circ - \theta$ or $\pi - \theta$	$180^\circ + \theta$ or $\pi + \theta$	$270^\circ - \theta$ or $\frac{3\pi}{2} - \theta$	$270^\circ + \theta$ or $\frac{3\pi}{2} + \theta$	$360^\circ - \theta$ or $2\pi - \theta$	$360^\circ + \theta$ or $2\pi + \theta$
Sine	$-\sin\theta$	$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$
Cosine	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$	$-\sin\theta$	$\cos\theta$
tangent	$-\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$	$-\cos\theta$	$\sin\theta$	$-\cos\theta$	$\cos\theta$
cotangent	$-\cot\theta$	$\tan\theta$	$-\tan\theta$	$-\cot\theta$	$\cot\theta$	$-\sin\theta$	$\cos\theta$	$-\sin\theta$	$\sin\theta$
secant	$\sec\theta$	$\operatorname{cosec}\theta$	$-\operatorname{cosec}\theta$	$-\sec\theta$	$-\sec\theta$	$-\operatorname{cosec}\theta$	$\operatorname{cosec}\theta$	$\sec\theta$	$\sec\theta$
cosecant	$-\operatorname{cosec}\theta$	$\sec\theta$	$\sec\theta$	$\operatorname{cosec}\theta$	$-\operatorname{cosec}\theta$	$-\sec\theta$	$\sec\theta$	$-\operatorname{cosec}\theta$	$\operatorname{cosec}\theta$

belongs to even group T function doesn't changes
If it belongs to odd group T ratio changes ($\sin\theta \leftrightarrow$

$$\cos\theta, \operatorname{cosec}\theta \leftrightarrow \sec\theta, \tan\theta \leftrightarrow \cot\theta$$

Step 4(Quadranting)

To assign the sign for the obtained value, follow the ASTC rule

Compound Angles Formulae:

Addition Formulae:

- $\sin(A + B) = \sin A \cos B + \cos A \sin B$
- $\cos(A + B) = \cos A \cos B - \sin A \sin B$
- $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

Subtraction Formulae:

- $\sin(A - B) = \sin A \cos B - \cos A \sin B$
- $\cos(A - B) = \cos A \cos B + \sin A \sin B$

$$3. \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Multiple Angles

1. $\sin 2A = 2 \sin A \cos A$
2. $\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$
3. $\cos 2A = \cos^2 A - \sin^2 A$
4. $\cos 2A = 2 \cos^2 A - 1$
5. $\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$
6. $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$
7. $\sin 3A = 3 \sin A - 4 \sin^3 A$
8. $\cos 3A = 4 \cos^3 A - 3 \cos A$
9. $\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$

Transformation of sum or difference into product

1. $\sin C + \sin D = 2 \sin \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$
2. $\sin C - \sin D = 2 \cos \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right)$
3. $\cos C + \cos D = 2 \cos \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$
4. $\cos C - \cos D = -2 \sin \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right)$

Transformation of product into sum or difference

1. $\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$
2. $\cos A \sin B = \frac{1}{2} [\sin(A+B) - \sin(A-B)]$
3. $\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$
4. $\sin A \sin B = \frac{1}{2} [\cos(A-B) - \cos(A+B)]$

Complex Numbers

Definition

A complex number is of the form $z = a + ib$, where:

a = Real part $\text{Re}(z)$

b = Imaginary part $\text{Im}(z)$

$$i = \sqrt{-1}$$

Example:

$$z = 3 + 4i \rightarrow \text{Re}(z) = 3, \text{Im}(z) = 4$$

Main values of i

$$i = \sqrt{-1}$$

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

Addition and Subtraction

$$(a + ib) + (c + id) = (a + c) + i(b + d)$$

$$(a + ib) - (c + id) = (a - c) + i(b - d)$$

$$\text{Example: } (3 + 4i) + (1 + 2i) = 4 + 6i$$

Multiplication

$$(a + ib)(c + id)$$

$$= (ac - bd) + i(ad + bc)$$

$$\text{Example: } (2 + 3i)(1 + 4i)$$

$$= 2 + 8i + 3i + 12i^2$$

$$= -10 + 11i$$

Conjugate of a Complex Number

if $z = a + ib$ then its

$$\text{Conjugate: } \bar{z} = a - ib$$

$$\text{Example: If } z = 3 + 4i \rightarrow \bar{z} = 3 - 4i$$

Modulus and Amplitude

$$\text{Modulus: } |z| = \sqrt{a^2 + b^2}$$

$$\text{Amplitude (Argument): } \alpha = \tan^{-1} \left(\left| \frac{b}{a} \right| \right)$$

But changes on different quadrants

Sign	Quadrant	
(+, +)	I	$\theta = \alpha$
(-, +)	II	$\theta = \pi - \alpha$
(-, -)	III	$\theta = -(\pi - \alpha)$
(+, -)	IV	$\theta = -\alpha$

$$\text{Example: For } z = 3 + 4i \rightarrow$$

$$|z| = \sqrt{3^2 + 4^2} = 5,$$

$$\theta = \tan^{-1}(4/3)$$

LIMITS

1. Constants and Variables

Constant: A fixed value (e.g., 5, -2, π).

Variable: A symbol that can take different values (e.g., x , y).

Function: A rule that assigns each input a unique output. Example: $f(x) = x^2 + 1$.

2. Concept of Limits

The limit of $f(x)$ as x approaches a is written as:

$$\lim f(x) = L$$

$$x \rightarrow a$$

This means as x gets closer to a , $f(x)$ gets closer to L .

3. Limits by Factorization Method

Steps: Factorize numerator/denominator and cancel common terms.

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$$

$$= \lim_{x \rightarrow a} \frac{(x-a)k}{(x-a)m}$$

Example: $\lim_{x \rightarrow 1} \frac{(x^2-1)}{(x-1)}$
 $= \lim_{x \rightarrow 1} \frac{(x-1)(x+1)}{(x-1)}$
 $= 2$

4. Limits by Rationalization Method

Multiply numerator and denominator by conjugate to simplify.

$$\lim_{x \rightarrow a} \frac{\sqrt{f(x)} - \sqrt{g(x)}}{x-a}$$

Multiply by $\sqrt{f(x)} + \sqrt{g(x)}$.

Example:

$$\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$$

Multiply numerator & denominator by $(\sqrt{x} + 2)$:

$$= \lim_{x \rightarrow 4} \frac{x-4}{(x-4)(\sqrt{x}+2)}$$

$$= \frac{1}{\sqrt{4}+2} = \frac{1}{4}$$

5. Limits at Infinity

When $x \rightarrow \infty$,

divide numerator and denominator by highest power of x .

Rule:

If degree(num) < degree(den) \rightarrow Limit = 0

If degree(num) = degree(den) \rightarrow Limit = ratio of coefficients

If degree(num) > degree(den) \rightarrow Limit = ∞

Example:

$$\lim_{x \rightarrow \infty} \frac{3x^2 + 5}{2x^2 - 7} = \frac{3}{2}$$

6. Standard limits

a) $\lim_{x \rightarrow a} \left(\frac{x^n - a^n}{x - a} \right) = na^{n-1}$, where n is rational

b) $\lim_{\theta \rightarrow 0} \left(\frac{\sin \theta}{\theta} \right) = 1$, where θ is in radians

c) $\lim_{\theta \rightarrow 0} \left(\frac{\tan \theta}{\theta} \right) = 1$ where θ is in radians

d) $\lim_{x \rightarrow 0} \left(\frac{e^x - 1}{x} \right) = 1$

Co - ordinate geometry

1. Slope of a straight line $m = \tan \theta$
2. Slope of line joining two points $m = \frac{y_2 - y_1}{x_2 - x_1}$
3. General form of equation of straight line $ax + by + C = 0$
4. Slope of a straight line $= -\frac{a}{b}$
 $X - \text{intercept} = -\frac{c}{a}$
 $Y - \text{intercept} = -\frac{c}{b}$
5. Slope intercept form $y = mx + C$
6. Two point form of a straight line $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$
7. Slope point form of a straight line $y - y_1 = m(x - x_1)$
8. Intercept form of the straight line $\frac{x}{a} + \frac{y}{b} = 1$
9. Equation of the straight line which is parallel to line $ax + by + c = 0$ and passing through the point (x_1, y_1) is $ax_1 + by_1 + K = 0$
10. Equation of the straight line which is perpendicular to the line $ax + by + c = 0$ and passing through the point (x_1, y_1) is $bx_1 - ay_1 + k = 0$
11. Angle between two lines is given by $\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$

DIFFERENTIAL CALUCUS

Derivatives of algebraic functions

1. $\frac{d}{dx}(x) = 1$
2. $\frac{d}{dx}(x^2) = 2x$
3. $\frac{d}{dx}(x^3) = 3x^2$
4. $\frac{d}{dx}(x^n) = nx^{n-1}$ where $n \in \mathbb{R}$
5. $\frac{d}{dx}\left(\frac{1}{x^n}\right) = -\frac{n}{x^{n+1}}$ where $n \in \mathbb{R}$
6. $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$
7. $\frac{d}{dx}\left(\frac{1}{x^2}\right) = -\frac{1}{x^3}$
8. $\frac{d}{dx}(k) = 0$ where k is constant
9. $\frac{d}{dx}(1) = 0$

$$10. \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$$

$$11. \frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = -\frac{1}{2x\sqrt{x}}$$

$$12. \frac{d}{dx}(ku) = k\left(\frac{d}{dx}\right)$$

Derivatives of trigonometric functions

$$13. \frac{d}{dx}(\sin x) = \cos x$$

$$14. \frac{d}{dx}(\cos x) = -\sin x$$

$$15. \frac{d}{dx}(\tan x) = \sec^2 x$$

$$16. \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$$

$$17. \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$18. \frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

Derivatives of Inverse trigonometric functions

$$19. \frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$20. \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$21. \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$22. \frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$$

$$23. \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$24. \frac{d}{dx}(\operatorname{cosec}^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$$

Derivatives of exponential functions

$$25. \frac{d}{dx}(a^x) = a^x \log a$$

$$26. \frac{d}{dx}(e^x) = e^x$$

Derivatives of logarithmic functions

$$27. \frac{d}{dx}(\log x) = \frac{1}{x}$$

Sum rule

Note: u, v, w are the functions of ' x '

$$28. \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

$$29. \frac{d}{dx}(u+v+w) = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx}$$

Product rule

$$30. \frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$31. \frac{d}{dx}(uvw) = uv \frac{dw}{dx} + vw \frac{du}{dx} + wu \frac{dv}{dx}$$

Quotient rule

$$32. \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu^1 - uv^1}{v^2} \text{ or}$$

$$\frac{d}{dx}\left(\frac{Nr}{Dr}\right) = \frac{Dr \frac{d(Nr)}{dx} - Nr \frac{d(Dr)}{dx}}{Dr^2}$$

where $Nr \rightarrow$ numerator, $Dr \rightarrow$ denominator

33. Slope of a tangent to the curve $y = f(x)$ at the point $p(x_1, y_1)$ is
 $m = \left(\frac{dy}{dx}\right)$

34. Equation of tangent to the curve $y = f(x)$ at the point $p(x_1, y_1)$ is
 $y - y_1 = m(x - x_1)$ where m is slope of a tangent

35. Slope of a normal to the curve $y = f(x)$ at the point $p(x_1, y_1)$ is $= -\frac{1}{m} = -\frac{1}{\left(\frac{dy}{dx}\right)}$

36. Equation of normal to the curve $y = f(x)$ at the point $p(x_1, y_1)$
 $y - y_1 = -\frac{1}{m}(x - x_1)$

INTEGRAL CALCULUS

$$1. \int 0 dx = C$$

$$2. \int 1 dx = x + C$$

$$3. \int k dx = kx + C$$

$$4. \int x^n dx = \frac{x^{n+1}}{n+1} + C; n \neq -1$$

$$5. \int \frac{1}{x^n} dx = -\frac{1}{(n-1)x^{n-1}} + c; n \neq 1$$

$$6. \int \frac{1}{x} dx = \ln |x| + C$$

$$7. \int \frac{1}{x^2} dx = -\frac{1}{x} + C$$

$$8. \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$$

$$9. \int \sqrt{x} dx = \frac{2}{3}x^{\frac{3}{2}} + C$$

$$10. \int a^x dx = \frac{a^x}{\log a} + C; a > 0, a \neq 1$$

$$11. \int e^x dx = e^x + C$$

$$12. \int \sin x dx = -\cos x + C$$

$$13. \int \cos x dx = \sin x + C$$

$$14. \int \tan x dx = \log \sec x + c$$

$$15. \int \cot x dx = \log \cos x + c$$

$$16. \int \sec x dx = \log(\sec x + \tan x) + c$$

$$17. \int \operatorname{cosec} x dx = \log(\operatorname{cosec} x - \cot x) + c$$

$$18. \int \sec^2 x dx = \tan x + C$$

$$19. \int \operatorname{cosec}^2 x dx = -\cot x + C$$

20. $\int \sec x (\tan x) dx = \sec x + C$
21. $\int \operatorname{cosec} x (\cot x) dx = -\operatorname{cosec} x + C$
22. $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$
23. $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$
24. $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + C$
25. $\int -\frac{1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C$
26. $\int -\frac{1}{1+x^2} dx = \cot^{-1} x + C$
27. $\int -\frac{1}{x\sqrt{x^2-1}} dx = \operatorname{cosec}^{-1} x + C$
28. $\int \sin(ax+b) dx = -\frac{\cos(ax+b)}{a} + C$
29. $\int \cos(ax+b) dx = \frac{\sin(ax+b)}{a} + C$
30. $\int e^{ax+b} dx = \frac{e^{ax+b}}{a} + C$
31. $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$
32. $\int \frac{1}{ax+b} dx = \frac{\log(ax+b)}{a} + C$

Sum rule

$$33. \int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$$

Difference rule

$$34. \int (f(x) - g(x)) dx = \int f(x) dx - \int g(x) dx$$

Product rule

$$35. \int uv dx = u \int v dx - \int \left(\int v dx \right) \frac{du}{dx} dx$$

where 'u' and 'v' are the functions of x

Definite integral

$$36. \text{ If } \int f(x) dx = \phi(x) \text{ then}$$

$$\int_a^b f(x) dx = [\phi(x)](b-a) = \phi(b) - \phi(a)$$

37. The area bounded by the curve $y = f(x)$, x-axis between the co-ordinates $x = a$ and $x = b$ is

$$\text{Area} = \int_a^b y dx = \int_a^b f(x) dx$$

38. The area bounded by the curve $y = f(x)$, y-axis between the co-ordinates $y = a$ and $y = b$ is

$$\text{Area} = \int_a^b x dy = \int_a^b g(y) dy$$

39. Volume of solid generated about x-axis is:

$$\text{Volume} = \pi \int_a^b y^2 dx$$

40. Volume of solid generated about y-axis is:

$$\text{Volume} = \pi \int_a^b x^2 dy$$